
pycontoltools Documentation

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Documentation for the code

lietools (package)

lietools (module)

The module **lietools** contains functions concerning different types of Lie-derivatives. It is based upon the *sympy* package for symbolic computation, especially the class *sympy.Matrix*.

`lietools.lietools.jac(expr, *args)`

Calculates the Jacobian matrix (derivative of a vectorial function) using the `jacobian()` function from the module `sympy.matrices.matrices.MatrixBase`.

Advantage: direct derivation of functions

Jacobian matrix:

$$J_f(a) := \begin{pmatrix} \frac{\partial f_1}{\partial x_1}(a) & \frac{\partial f_1}{\partial x_2}(a) & \dots & \frac{\partial f_1}{\partial x_n}(a) \\ \vdots & \vdots & \ddots & \vdots \\ \frac{\partial f_m}{\partial x_1}(a) & \frac{\partial f_m}{\partial x_2}(a) & \dots & \frac{\partial f_m}{\partial x_n}(a) \end{pmatrix}$$

Parameters

- expr** [expression to derive] function / row matrix/ column matrix
- args** [coordinates] separate or as list-like object

Return

- returns : Jacobi matrix
- type : `sympy.Matrix`

Examples

```
>>> import sympy
>>> x1,x2,x3 = sympy.symbols('x1 x2 x3')
```

```
>>> jac(x1**2+2*x2+x3, x1, x2, x3)
Matrix([[2*x1, 2, 1]])
```

See also:

`sympy.jacobian()`

`lietools.lietools.lie_deriv(sf,vf,x,n=1)`

Calculates the Lie derivative of a scalar field $\lambda(x)$ along a vector field $f(x)$ (e.g. [\[Isidori\]](#)):

$$L_f\lambda(x) = \frac{\partial\lambda(x)}{\partial x}f(x) = \text{grad}^T\lambda(x) \cdot f(x)$$

with $L_f^n\lambda(x) = \frac{\partial L_f^{n-1}\lambda(x)}{\partial x}f(x)$ and $L_f^0\lambda(x) := \lambda(x)$

Parameters

- sf** [scalar field to be derived] function
- vf** [vector field to derive along] vector
- x** [coordinates for derivation] list
- n** [number of derivations] non-negative integer

Return

- returns : scalar field
- type : function

Examples

```
>>> import sympy
```

```
>>> x1,x2,x3 = sympy.symbols('x1 x2 x3')
>>> h = x1**2 + 2*x2 + x3
>>> f = sympy.Matrix([x1*x2, x3, x2])
>>> x = [x1, x2, x3]
```

```
>>> lie_deriv(h, f, x, n=1)
2*x1**2*x2 + x2 + 2*x3
```

See also:

`lie_bracket(), lie_deriv_covf()`

`lietools.lietools.lie_bracket(f,g,*args,**kwargs)`

Calculates the Lie bracket for the vector field $g(x)$ along the vector field $f(x)$ (e.g. [\[Isidori\]](#)):

$$[f,g] = \frac{\partial g(x)}{\partial x}f(x) - \frac{\partial f(x)}{\partial x}g(x) = \text{ad}_f g(x)$$

with $ad_f^n g(x) = [f, ad_f^{n-1} g](x)$ and $ad_f^0 g(x) := g(x)$

Parameters

- **f** [vector field (direction for derivation)] Matrix (shape: (n, 1)) / iterable
- **g** [vector field to be derived] Matrix (shape: (n, 1)) / iterable
- **args** [coordinates] separate scalar symbols or as iterable

Keyword Arguments

- **n** [number of derivations] non-negative integer (default = 1)

Exceptions

- `AssertionError` : non-matching shapes of f, g, args

Return

- returns : vector field
- type : Matrix

Examples

```
>>> import sympy
```

```
>>> x1,x2,x3 = sympy.symbols('x1 x2 x3')
>>> g = [2*x2, x1**2, 2*x3]
>>> f = [x1*x2, x3, x2]
```

```
>>> lie_bracket(g, f, x1, x2, x3, n=1)
Matrix([
[x1**3 + 2*x2**2 - 2*x3],
[ -2*x1**2*x2 + 2*x3],
[      x1**2 - 2*x2]])
```

See also:

`lie_deriv()`, `lie_deriv_covf()`

`lietools.lietools.lie_deriv_covf(w,f,*args,**kwargs)`

Calculates the Lie derivative of the covector field $\omega(x)$ along the vector field $f(x)$ (e.g. [Isidori]):

$$L_f \omega(x) = f^T(x) \left(\frac{\partial \omega^T(x)}{\partial x} \right)^T + \omega(x) \frac{\partial f(x)}{\partial x}$$

with

$$L_f^n \omega(x) = f^T(x) \left(\frac{\partial (L_f^{n-1} \omega)^T(x)}{\partial x} \right)^T + (L_f^{n-1} \omega)(x) \frac{\partial f(x)}{\partial x}$$

and $L_f^0 \omega(x) := \omega(x)$

Includes the option to omit the transposition of $\frac{\partial \omega^T(x)}{\partial x}$ with `transpose_jac = False`:

$$L_f \omega(x) = f^T(x) \left(\frac{\partial \omega^T(x)}{\partial x} \right) + \omega(x) \frac{\partial f(x)}{\partial x}$$

Parameters

- **w** [covector field to be derived] Matrix of shape (1,n)
- **f** [vector field (direction of derivation)] Matrix of shape (n,1)
- **args** [coordinates] separate or as list-like object

Keyword Arguments

- **n** [number of derivations] non-negative integer (default = 1)
- **transpose_jac** [transposition of $\frac{\partial \omega^T(x)}{\partial x}$] boolean (default = True)(Background: needed for some special applications)

Exceptions

- **AssertionError** : non-matching shapes of w, f, args

Return

- **returns** : covector field
- **type** : sympy.Matrix

Examples

```
>>> import sympy
```

```
>>> x1,x2,x3 = sympy.symbols('x1 x2 x3')
>>> w = sympy.Matrix([[2*x2, x1**2, 2*x3]])
>>> f = sympy.Matrix([x1, x2, x3])
```

```
>>> lie_deriv_covf(w, f, x1, x2, x3, n=1)
Matrix([[4*x2, 3*x1**2, 4*x3]])
```

See also:

```
lie_deriv(), lie_bracket()
```

linearcontrol

linearcontrol

The module **linearcontrol** contains functions concerning linear control algorithms.

```
linearcontrol.linearcontrol.cont_mat(A, B)
    Kallmanns controlability matrix
```

```
linearcontrol.linearcontrol.is_left_coprime(Ap, Bp=None, eps=1e-10)
    Test ob Ap,Bp Linksteilerfrei sind keine Parameter zulässig
```

```
linearcontrol.linearcontrol.linear_input_trafo(B, row_idcs)
    serves to decouple inputs from each other
```


robust_poleplacement

The module **robust_poleplacement** contains functions to calculate a robust control matrix for multiple input systems.

`linearcontrol.robust_poleplacement.exchange_all_cols(V, P_list)`

For every column in V : Calculates the 1-dimensional basis of the annihilator ($:= a_j$) of all the other columns in V and projects a_j to its correspondent space out of P_list .

Then in V : replaces v_j with the new normalized projected vector $v_{j,projected}$.

See also:

`opt_place_MI()`

Parameters

- **V** [matrix of eigenvectors $V = (v_1, \dots, v_n)$] `sympy.Matrix`
- **P_list** [list of spaces $(S_h)_i$ for $i \in (1, \dots, n)$] list

Return

- returns : new eigenvector matrix V
- type : `sympy.Matrix`

`linearcontrol.robust_poleplacement.full_qr(A, only_null_space=False)`

Performs the QR numpy decomposition and augments the reduced orthonormal matrix Q_{red} by its transposed null space $\text{null}(Q_{red})^T$ (such that Q is quadratic and regular).

$$Q = (Q_{red} \quad \text{null}(Q_{red})^T)$$

Parameters

- **A** [matrix to be QR decomposed] `sympy.Matrix`

Keyword Arguments

- **only_null_space** [only the null space of Q_{red} will be returned] boolean (default = False)

Return

- returns : Q (quadratic & regular)
- type : `sympy.Matrix`
- returns : r (upper triangular matrix)
- type : `sympy.Matrix`

`linearcontrol.robust_poleplacement.opt_place_MI(A, B, *eigenvals, **kwargs)`

Calculates and returns the optimal control matrix B_K for the new system matrix $(A + BB_K)$ of the closed loop system by the algorithm described in [\[Reinschke14\]](#).

Parameters

- **A** [state matrix of the open loop] `sympy.Matrix`
- **B** [input matrix] `sympy.Matrix`
- **eigenvals** [desired eigenvalues for the closed loop system] separate or as list-like object

Keyword Arguments

- rtol** [relative tolerance of the change in the last iteration step of the] resulting determinant of the eigen-vector matrix
real number (default = 0.01)

Return

- returns : B_K
- type : sympy.Matrix

`linearcontrol.robust_poleplacement.ortho_complement(M)`

Gets a n,n-matrix M which is assumed to have rank n-1 and returns a “column” v with $v^T M = 0$ and $v^T v = 1$.

Parameters

- M** [matrix of vectors with rank n-1] sympy.Matrix

Return

- returns : orthogonal complement for columns of M
- type : numpy.array (1d)

trajectories

trajectories

The module **trajectories** contains functions concerning the construction of system trajectories.

`trajectories.trajectories.integrate_pw(fnc, var, transpoints)`

due to a bug in sympy we must correct the offset in the integral to make the result continuous

`trajectories.trajectories.make_pw(var, transpoints, fncs)`

auxfuncs/math

The auxfuncs package **math** contains mathematical auxiliary functions for **pycontroltools** categorized in the modules concerning:

- differential operators
- LaPlace
- matrices
- miscellaneous
- numerical tools
- polynomial helpfunctions
- Taylor

diffoperators

The module **diffoperators** contains functions concerning differential Operators.

`auxfuncs.math.diffoperators.div(vf, x)`

divergence of a vector field

`auxfuncs.math.diffoperators.gradient` (*scalar_field*, *xx*)
 # returns a row vector (coverctorfiel)!

`auxfuncs.math.diffoperators.hoderiv` (*f*, *x*, *N=2*)
 computes a Higher Order derivative of the vectorfield *f*

Result is a tensor of type (N,0)

or a $n \times L \times \dots \times L$ (N times) hyper Matrix

(represented a (N+1)-dimensional numpy array

laplace

The module **laplace** contains functions concerning LaPlace.

`auxfuncs.math.laplace.do_laplace_deriv` (*laplace_expr*, *s*, *t*)
 Example: `laplace_expr = s*(t**3+7*t**2-2*t+4)` returns: $3*t**2 + 14*t - 2$

matrix

The module **matrix** contains functions concerning operations on matrices.

`auxfuncs.math.matrix.all_k_minors` (*M*, *k*, ***kwargs*)
 returns all minors of order *k* of *M*

Note that if `k == M.shape[0]`

this computes all “column-minors”

`auxfuncs.math.matrix.as_mutable_matrix` (*matrix*)
 sympy sometimes converts matrices to immutable objects this can be reverted by a call to `.as_mutable()` this function provides access to that call as a function (just for cleaner syntax)

`auxfuncs.math.matrix.cancel_rows_cols` (*M*, *rows*, *cols*)
 cancel rows and cols form a matrix

rows ... rows to be canceled cols ... cols to be canceled

`auxfuncs.math.matrix.col_degree` (*col*, *symb*)

`auxfuncs.math.matrix.col_minor` (*A*, **cols*, ***kwargs*)
 returns the minor (determinant) of the columns in *cols*

`auxfuncs.math.matrix.col_select` (*A*, **cols*)
 selects some columns from a matrix

`auxfuncs.math.matrix.col_stack` (**args*)
 takes some col vectors and aggregates them to a matrix

`auxfuncs.math.matrix.concat_cols` (**args*)
 takes some col vectors and aggregates them to a matrix

`auxfuncs.math.matrix.concat_rows` (**args*)
 takes some row (hyper-)vectors and aggregates them to a matrix

`auxfuncs.math.matrix.elementwise_mul` (*M1*, *M2*)
 performs element wise multiplication of matrices

`auxfuncs.math.matrix.ensure_mutable` (*arg*)
 ensures that we handle a mutable matrix (iff *arg* is a matrix)

`auxfuncs.math.matrix.expand(arg)`
`sp.expand` currently has no matrix support

`auxfuncs.math.matrix.general_minor(A, rows, cols, **kwargs)`
 selects some rows and some cols of A and returns the det of the resulting Matrix

`auxfuncs.math.matrix.getOccupation(M)`
 maps ($m_{ij} \neq 0$) to every element

`auxfuncs.math.matrix.get_col_reduced_right(A, symb, T=None, return_internals=False)`
 Takes a polynomial matrix A(s) and returns a unimod Transformation T(s) such that $A(s) \cdot T(s)$ (i.e. right multiplication) is col_reduced.

Approach is taken from appendix of the PHD-Thesis of S. O. Lindert (2009)

Args A: Matrix s: symbol T: unimod-Matrix from preceeding steps

-> recursive approach

Returns Ar: reduced Matrix T: unimodular transformation Matrix

`auxfuncs.math.matrix.get_rows(A)`
 returns a list of $n \times 1$ vectors

`auxfuncs.math.matrix.is_col_reduced(A, symb, return_internals=False)`
 tests whether polynomial Matrix A is column-reduced

optionally returns internal variables: the list of col-wise max degrees the matrix with the col.-wise-highest coeffs (Gamma)

Note: concept of column-reduced matrix is important e.g. for solving a Polynomial System w.r.t. highest order “derivative”

Note: every matrix can be made col-reduced by unimodular transformation

`auxfuncs.math.matrix.is_row_reduced(A, symb, *args, **kwargs)`
 transposed Version of `is_col_reduced(...)`

`auxfuncs.math.matrix.matrix_atoms(M, *args, **kwargs)`

`auxfuncs.math.matrix.matrix_count_ops(M, visual=False)`

`auxfuncs.math.matrix.matrix_degrees(A, symb)`

`auxfuncs.math.matrix.matrix_random_equalitytest(M1, M2, info=False, **kwargs)`

`auxfuncs.math.matrix.matrix_series(m, xx, order, poly=False)`

`auxfuncs.math.matrix.matrix_subs_random_numbers(M)`
 substitute every symbol in M with a random number

this might be usefull to determine the generic rank of a matrix

`auxfuncs.math.matrix.matrix_with_rationals(A)`

`auxfuncs.math.matrix.mdifff(M, var)`
 returns the elementwise derivative of a matrix M w.r.t. var

`auxfuncs.math.matrix.ratsimp(arg)`
`sp.ratsimp` currently has no matrix support

`auxfuncs.math.matrix.row_col_select(A, rows, cols)`
 selects some rows and some cols of A and returns the resulting Matrix

`auxfuncs.math.matrix.row_stack(*args)`
 takes some row (hyper-)vectors and aggregetes them to a matrix

```
auxfuncs.math.matrix.simplify(arg)
    sp.simplify currently has no matrix support

auxfuncs.math.matrix.symbMatrix(n, m, s='a', symmetric=0)

auxfuncs.math.matrix.symm_matrix_to_vect(M)
    converts a b b c
        to [a, b, c]

auxfuncs.math.matrix.symmetryDict(M)
    erstellt ein dict, was aus einer beliebigen Matrix M mittels M.subs(..) eine symmetrische Matrix macht

auxfuncs.math.matrix.trigsimp(arg, **kwargs)
    sp.trigsimp currently has no matrix support

auxfuncs.math.matrix.unimod_completion(col, symb)
    takes a column and completes it such that the result is unimodular

auxfuncs.math.matrix.vect_to_symm_matrix(v)
    converts [a, b, c]
        to a b b c
```

miscmath

The module **miscmath** contains miscellaneous mathematical functions for **pycontroltools**.

```
class auxfuncs.math.miscmath.equation(lhs, rhs=0)
    #chris: Klasse equation erstellt Gleichungs-Objekte mittels sympify mit Attributen für Lefthandside (lhs) und
    Righthandside (rhs) der Gleichung

auxfuncs.math.miscmath.extract_independent_eqns(M)
    handles only homogeneous eqns

    M Matrix

    returns two lists: indices_of_rows, indices_of_cols

auxfuncs.math.miscmath.fractionfromfloat(x_, maxden=1000)
    fraction from float args:

    x maxdenominator (default = 1000)

auxfuncs.math.miscmath.get_coeff_row(eq, vars)
    takes one equation object and returns the corresponding row of the system matrix

auxfuncs.math.miscmath.jac(expr, *args)
    Calculates the Jacobian matrix (derivative of a vectorial function) using the jacobian() function from the
    module sympy.matrices.matrices.MatrixBase.

Advantage: direct derivation of functions

Jacobian matrix:
```

$$J_f(a) := \begin{pmatrix} \frac{\partial f_1}{\partial x_1}(a) & \frac{\partial f_1}{\partial x_2}(a) & \dots & \frac{\partial f_1}{\partial x_n}(a) \\ \vdots & \vdots & \ddots & \vdots \\ \frac{\partial f_m}{\partial x_1}(a) & \frac{\partial f_m}{\partial x_2}(a) & \dots & \frac{\partial f_m}{\partial x_n}(a) \end{pmatrix}$$

Parameters

- expr** [expression to derive] function / row matrix/ column matrix
- args** [coordinates] separate or as list-like object

Return

- returns : Jacobi matrix
- type : sympy.Matrix

Examples

```
>>> import sympy
>>> x1,x2,x3 = sympy.symbols('x1 x2 x3')
```

```
>>> jac(x1**2+2*x2+x3, x1, x2, x3)
Matrix([[2*x1, 2, 1]])
```

See also:

`sympy.jacobian()`

`auxfuncs.math.miscmath.lin_solve_all(eqns)`

takes a list of equations and tries to solve wrt. to all occurring symbols

`auxfuncs.math.miscmath.lin_solve_eqns(eqns, vars)`

takes a list of equation objects creates a system matrix of and calls `sp.solve`

`auxfuncs.math.miscmath.lin_solve_eqns_jac(eqns, vars)`

takes a list of equation objects creates a system matrix of and calls `sp.solve`

new version !! # should replace `lin_solve_eqns`

assumes that `eqns` is a list of expressions where `rhs = 0`

`auxfuncs.math.miscmath.make_eqns(v1, v2=None)`

#chris: mehrere lhs,rhs übergeben und daraus Gleichungen erstellen

`auxfuncs.math.miscmath.multi_series(expr, xx, order, poly=False)`

Reihenentwicklung (um 0) eines Ausdrucks in mehreren Variablen

`auxfuncs.math.miscmath.numer_denom(expr)`

`auxfuncs.math.miscmath.rat_if_close(x, tol=1e-10)`

`auxfuncs.math.miscmath.rationalize_expression(expr, tol=1e-10)`

substitutes real numbers occuring in `expr` which are closer than `tol` to a rational with a sufficiently small denominator with these rationals

usefull special case `1.2346294e-15 -> 0`

`auxfuncs.math.miscmath.real_roots(expr)`

`auxfuncs.math.miscmath.roots(expr)`

`auxfuncs.math.miscmath.sp_fff(x, maxden)`

`sympy_fraction` from float #chris: nimmt anscheinend Objekte vom Typ `fractions.Fraction`

(`Fraction(133, 10)`) und stellt sie als Bruch dar (`133/10`)

`auxfuncs.math.miscmath.syms_to_func(expr, syms, arg)`

in `expr` replace `x` by `x(arg)` where `x` is any element of `syms`

`auxfuncs.math.miscmath.trigsimp2(expr)`

`sin**2 + cos**2 = 1` in big expressions

`auxfuncs.math.miscmath.uv(n, i)`
unit vectors (columns)

numtools

The module **numtools** contains numerical tools.

`auxfuncs.math.numtools.chop(expr, tol=1e-10)`
suppress small numerical values

`auxfuncs.math.numtools.clean_numbers(expr, eps=1e-10)`
trys to clean all numbers from numeric noise

`auxfuncs.math.numtools.cont_continuation(x, stephight, threshold)`
continuous continuation (for 1d-arrays)

`x data`

`stephight ... the expected stephight (e.g 2*pi)`

threshold smallest difference which is considered as a discontinuity which has to be corrected (must be greater than the Lipschitz-Const. of the signal times dt)

`auxfuncs.math.numtools.dd(a, b, c, ...) = np.dot(a, np.dot(b, np.dot(c, ...)))`

`auxfuncs.math.numtools.extrema(x, max=True, min=True, strict=False, withend=False)`
This function will index the extrema of a given array x.

Options: max If true, will index maxima min If true, will index minima strict If true, will not index changes to zero gradient withend If true, always include x[0] and x[-1]

This function will return a tuple of extrema indexies and values

`auxfuncs.math.numtools.np_trunc_small_values(arr, lim=1e-10)`

`auxfuncs.math.numtools.null(A, eps=1e-10)`
null-space of a Matrix or 2d-array

`auxfuncs.math.numtools.pyc2d(a, b, Ts)`
Algorithmus kopiert von Roberto Bucher

Begründung: man erweitert den Zustand $x_{neu} = (x, u)$ und sagt $u_{dot} = 0$ (weil $u = konst.$) Für das neue System bekommt man die zusammengestzte Matrix und pflückt sie hinterher wieder auseinander.

`auxfuncs.math.numtools.random_equaltest(exp1, exp2, info=False, integer=False, seed=None, tol=1e-14, min=-1, max=1)`
serves to check numerically (with random numbers) whether exp1, ep2 are equal # TODO: unit test

`auxfuncs.math.numtools.to_np(arr, dtype=<type 'float'>)`
converts a sympy matrix in a nice numpy array

`auxfuncs.math.numtools.zero_crossing_simulation(rhs, zcf, z0, t_values)`
scipy.odeint does not provide a zero crossing function naive (and slow) approach

`rhs:` rhs function `zcf:` the function whose zerocrossing shall be detected

takes the state (shape =(n,m) returns shape=n

`z0:` initial state `t_values:` time values (up to which the zc event is suspected)

polynomial

The module **polynomial** contains functions concerning the construction of polynomials.

```
auxfuncs.math.polynomial.coeffs (expr, var=None)
    if var == None, assumes that there is only one variable in expr

auxfuncs.math.polynomial.condition_poly (var, *conditions)
    # this function is intended to be a generalization of trans_poly

    returns a polynomial y(t) that fullfills given conditions

    every condition is a tuple of the following form:

    (t1, y1, *derivs) # derivs contains cn derivatives

    every derivative (to the highest specified [in each condition]) must be given

auxfuncs.math.polynomial.element_deg_factory (symb)
    returns a function for getting the polynomial degree of an expr. w.r.t. a certain symbol

auxfuncs.math.polynomial.get_order_coeff_from_expr (expr, symb, order)

    example: 3*s**2 -4*s + 5, s, 3 -> 0 3*s**2 -4*s + 5, s, 2 -> 3 3*s**2 -4*s + 5, s, 1 -> -4 3*s**2 -4*s + 5, s, 9
    -> 0

auxfuncs.math.polynomial.poly_coeffs (expr, var=None)
    returns all (monovariate)-poly-coeffs (including 0s) as a list first element is highest coeff.

auxfuncs.math.polynomial.poly_degree (expr, var=None)
    returns degree of monovariate polynomial

auxfuncs.math.polynomial.poly_scalar_field (xx, symbgen, order, poly=False)
    returns a multivariate poly with specified orders and symbolic coeffs returns also a list of the coefficients

auxfuncs.math.polynomial.trans_poly (var, cn, left, right)
    returns a polynomial y(t) that is cn times continuous differentiable

    left and right are sequences of conditions for the boundaries

    left = (t1, y1, *derivs) # derivs contains cn derivatives

auxfuncs.math.polynomial.zeros_to_coeffs (*z_list, **kwargs)
    calculates the coeffs corresponding to a poly with provided zeros
```

taylor

The module **taylor** contains functions concerning the construction of Taylor polynomials.

```
auxfuncs.math.taylor.multi_taylor (expr, args, x0=None, order=1)
    compute a multivariate taylor polynomial of a scalar function

    default: linearization about 0 (all args)

auxfuncs.math.taylor.multi_taylor_matrix (M, args, x0=None, order=1)
    applies multi_taylor to each element

auxfuncs.math.taylor.series (expr, var, order)
    taylor expansion at zero (without O(.))
```


auxfuncs/programming

The helpfunctions package **programming** contains helpfunctions for **pycontroltools** concerning programming issues.

miscprog

The module **miscprog** contains miscellaneous functions concerning programming in **pycontroltools**.

`auxfuncs.programming.miscprog.atoms (expr, *args, **kwargs)`

`auxfuncs.programming.miscprog.aux_make_tup_if_necc (arg)`
checks whether arg is iterable. if not return (arg,)

`auxfuncs.programming.miscprog.expr_to_func (args, expr, modules='numpy', **kwargs)`
wrapper for sympy.lambdify to handle constant expressions (shall return a numpyfied function as well)

this function bypasses the following problem:

```
f1 = sp.lambdify(t, 5*t, modules = "numpy") f2 = sp.lambdify(t, 0*t, modules = "numpy")
```

```
f1(np.arange(5)).shape # -> array f2(np.arange(5)).shape # -> int
```

Some special kwargs: `np_wrapper == True`:

the return-value of the resulting function is passed through to `np(..)` before returning

`auxfuncs.programming.miscprog.get_diffterms (xx, order)`
returns a list such as

```
[(x1, x1), (x1, x2), (x1, x3), (x2, x2), (x2, x3), (x3, x3)]
```

for `xx = (x1, x2, x3)` and `order = 2`

`auxfuncs.programming.miscprog.get_expr_var (expr, var=None)`
auxillary function if `var == None` returns the unique symbol which is contained in `expr`: if no symbol is found, returns `None`

`auxfuncs.programming.miscprog.makeGlobal (varList)`
injects the symbolic variables of a collection to the global namespace usefull for interactive sessions

`auxfuncs.programming.miscprog.make_global (varList)`
injects the symbolic variables of a collection to the global namespace usefull for interactive sessions

`auxfuncs.programming.miscprog.prev (expr, **kwargs)`
sympy preview abbreviation

`auxfuncs.programming.miscprog.rev_tuple (tup)`

`auxfuncs.programming.miscprog.simp_trig_dict (sdict)`
takes a sorted dict, simplifies each value and adds all up

`auxfuncs.programming.miscprog.subs_same_syms (x+y[, x, y])`
returns `x+y`, where the symbols are taken from the list (syms in `exp` might be different objects with the same name)

this functions helps if `expr` comes from a string

`auxfuncs.programming.miscprog.trig_term_poly (expr, s)`
`s ...` the argument of `sin`, `cos`

`auxfuncs.programming.miscprog.tup0 (xx)`
helper function for substituting. takes `(x1, x2, x3, ...)` returns `[(x1, 0), (x2, 0), ...]`

`auxfuncs.programming.miscprog.zip0 (xx, arg=0)`
handy for substituting equilibrium points

How to build the Sphinx documentation for pycontroltools

The documentation for the **pycontroltools** module is created using Sphinx. To be able to use the tools Sphinx has to be installed (<http://sphinx-doc.org/latest/install.html>) and initialized.

Initializing Sphinx

To initialize the Sphinx folder hierarchy for the project, you preferably create a new folder for the documentation in the main folder of your project. (The script files containing the docstrings have to be in the same directory as the documentation folder or in the documentation folder itself.)

Now start a command console in that folder and type:

```
sphinx-quickstart
```

Build HTML- and LaTeX-Files

Copy and replace the files from the **pycontroltools_sphinx** folder of the repository into your documentation folder.

To build the HTML-files type:

```
make html
```

The build-files appear in the folder **_build/html**.

To build the LaTeX-Files type:

```
make latex
```

The build-files appear in the folder **_build/latex**. If a LaTeX distribution is installed on your system, you can now use the LaTeX files to build (and edit) the document with your own LaTeX editor.

CHAPTER 2

Source materials:

Source materials

lie

linearcontrol

CHAPTER 3

Indices and tables

- `genindex`
- `modindex`
- *Source materials*
- `search`

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